647510 **MSC3MatC304x** **Seat No :________**

M.Sc.(Maths.) Semester - 3 (*CBCS*) Examination

Oct/Nov-2021 (NEW COURSE)

Discrete Mathematics (Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1 (a) **4 Marks**

State and Prove Langrange's Theorem.

Q.1 (b) **Answer any two** **10 Marks**

(1) State and prove Fundamental Theorem of Homomorphism of Groups.

(2) Define Transitive Closure. State and Prove necessary and sufficient condition for Transitive closure.

(3) Prove that every monoid has Unique identity and unique inverse.

Q.2 (a) **4 Marks**

Define Homomorphism of lattices. Prove that (S_6, D) & (S_{30}, D) are homomorphic.

Q.2 (b) **Answer any two** **10 Marks**

(1) Using K map minimize the following Boolean Expression

$$\alpha\,(x, y, z, t) = x\,y' + x\,y\,z + x'y'z' + x'y\,z\,t$$

(2) Define Boolean Function. Find the value of α (0,0,1) for the Boolean expression $\alpha\,(x, y, z) = \Sigma$ (0,1,2,5)

(3) Express the following as equivalent of PoS & SoP Canonical Form for Boolean Expression $\alpha\,(x, y, z) = x' + y\,z'$

Q.3 (a) **4 Marks**

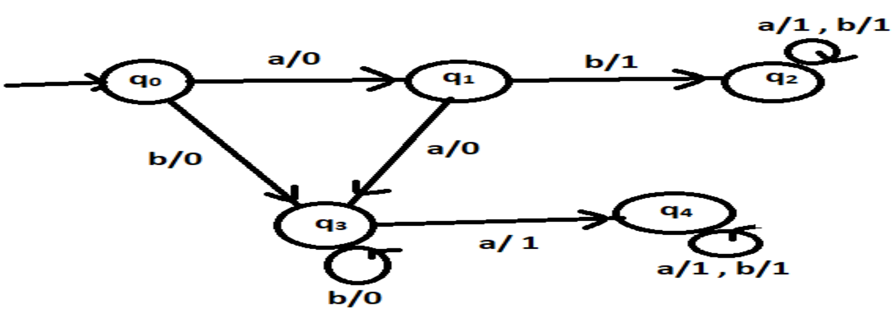
State and Prove Pumping Lemma.

Q.3 (b) **Answer any two.** **10 Marks**

(1) Define Finite state machine. FSM M is given by transition diagram.

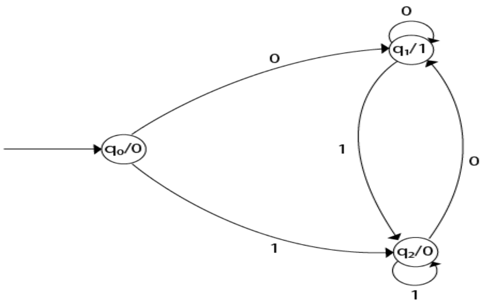
(1) Determine the machine

(2) Construct input –output table. Check whether the string is accepted by the machine? String: aabb. (3) Draw directed walk of the string.



(2) What is language? Write down operations on languages.

(3) Define Moore machine. Find the output of the string 1011 in the following Moore machine.



Q.4 (a) **4 Marks**

Define Conjunction connectives. State and prove absorption law for logical connectives.

Q.4 (b) Answer any two **10 Marks**

(1) Define Contradiction. Prove the following statement formula is tutology ,using properties of logical connnectives:
$(p \rightarrow q) \rightarrow (\sim q \rightarrow \sim p)$

(2) Check the validity of the following argument:
If I study, then I will not fail mathematics.If I do not play basketball, then I will study.But I failed mathematics.Therefore I must have played basketball.

(3) Define n-place predicate. Check the validity of the argument using predicate and quantifiers. "Everyone who knows english well then get a good job. Ram did not get the good job.So he did not know English well."

Q.5 (a) **4 Marks**

For any block code with minimum Hamming distance at least 2t + 1 between code words, show that:

$$2^{n-k} \geq 1 + \binom{n}{1} + \binom{n}{2} + \ldots \binom{n}{t}.$$

Q.5 (b) Answer any two **10 Marks**

(1) Let C be Binary(6,3) code with parity check matrix

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

(1) Find the cost leaders and their syndromes.
(2) Use syndrome decoding to decode the received vector: 110000

(2) Let C be the binary linear code with generator matrix

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Write down a parity check matrix H for C. Explain how the minimum distance of C may be deduced from H. Find d(C).

(3) Find the generator matrix of C. The parity check matrix is simply obtained from H as,

$$G = [A^T \cdot I_3] = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

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